Engineering an atom-interferometer with modulated light-induced 3π spin-orbit coupling

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(Dated: February 18, 2015)

We have developed an experimental method to modify the single-particle dispersion using periodic modulation of Raman beams which couple two spin-states of an ultracold atomic gas. The modulation introduces a new coupling between Raman-induced spin-orbit-coupled dressed bands, creating a second generation of dressed-state eigenlevels that feature both a novel 3π spin-orbit coupling and a pair of avoided crossings, which is used to realize an atomic interferometer. The spin polarization and energies of these eigenlevels are characterized by studying the transport of a Bose-Einstein condensate in this system, including observing a Stueckelberg interference.

PACS numbers: 03.75.Lm, 03.75.Dg, 67.85.De

In ultracold atoms, laser-induced synthetic gauge fields have been used to modify the single-particle dispersion and realize physics such as synthetic magnetic fields [1], spin-orbit (SO) coupling [2] and the Spin-Hall effect [3], the Hofstadter/Harper and Haldane Hamiltonians [4–6]. Raman-coupling between spin states of ultracold atoms which modifies the free-particle dispersion relation [7] has been the basis of many of these works [8, 9], and has resulted in a rich field of studies in Rashba-Dresselhaus type SO coupling for both Bose-Einstein condensates (BECs) and degenerate Fermi gases [10–18].

In this work, we investigate adding a time-dependent modulation to the intensity of the Raman-coupling as a new way to engineer the dispersion relations of ultracold atoms. We demonstrate two results to show the effectiveness of this technique: (1) by observing a 3π rotation of the spin polarization in the spin-momentum locked dressed eigenlevels and dispersion relation, which we call a 3π SO coupling, and (2) using the eigenlevels in the dressed state picture of the modulated Raman system to create an atom interferometer.

Our experiment starts with using SO coupling of equal parts Rashba and Dresselhaus that is created by Raman coupling two spin states of an atom [2], shown in Fig. 1 (a) with a similar experimental setup to our previous work (see Ref. [17]). This SO coupling consists of two eigenlevels, $E_U(q)$ and $E_L(q)$ for the upper and lower, both of which possess a spin-momentum "locking" in quasimomentum, $\hbar q$, space. We then create an inter-eigenlevel coupling by modulating the strength of the Raman coupling: $\Omega_R(t) = \Omega_0 + \Omega_M cos(2\pi f_{mod} t)$, where f_{mod} is the modulation frequency, Ω_0 is the unmodulated Raman coupling, and Ω_M is the modulation amplitude.

In the theoretical analysis of this system, we first calculate the Rashba-Dresselhaus SO coupled eigenlevels from the Hamiltonian:

$$\mathcal{H}_{SO} = \begin{pmatrix} \frac{\hbar^2}{2m}(q+k_r)^2 - \delta/2 & \Omega_0/2 \\ \Omega_0/2 & \frac{\hbar^2}{2m}(q-k_r)^2 + \delta/2 \end{pmatrix} \quad (1)$$

to find the SO eigenlevel structure, as pictured in Fig. 1 (a) where δ is the Raman laser detuning, m is the atomic mass, k_r is the recoil momentum, and $\hbar$ is the reduced Planck's constant ($h = 2\pi\hbar$). The third spin state is neglected due to the quadratic Zeeman shift. We define total atom spin polarization as $\mathcal{S} = (N_\downarrow - N_\uparrow)/(N_\downarrow + N_\uparrow)$, where $N_{\uparrow(\downarrow)}$ is the number of spin up (down) atoms in the BEC.

Using the dressed state picture, these original SO eigenlevel structure ($E_U(q)$ and $E_L(q)$) forms the basis of the manifold which is replicated in every hf_{mod} to form a set $E_{U/L,n} = E_{U/L} + nhf_{mod}$ where $n = 0, \pm1, \pm2, ...$ in the modulated system. Considering just nearest neighbor eigenlevels ($n = \pm1$) with a modulation frequency slightly larger than the eigenlevel energy difference near $q = 0$ ($hf_{mod} \sim E_U(q \approx 0) - E_L(q \approx 0)$), the ground eigenlevel $E_{L,n=0}$ has a pair of crossings with $E_{U,n=-1}$ (the precise value of f_{mod} determines the locations of the crossings in quasi-momentum space). We use these eigenlevels as the new bare states coupled by the modulation field with an effective coupling strength denoted Ω_C.

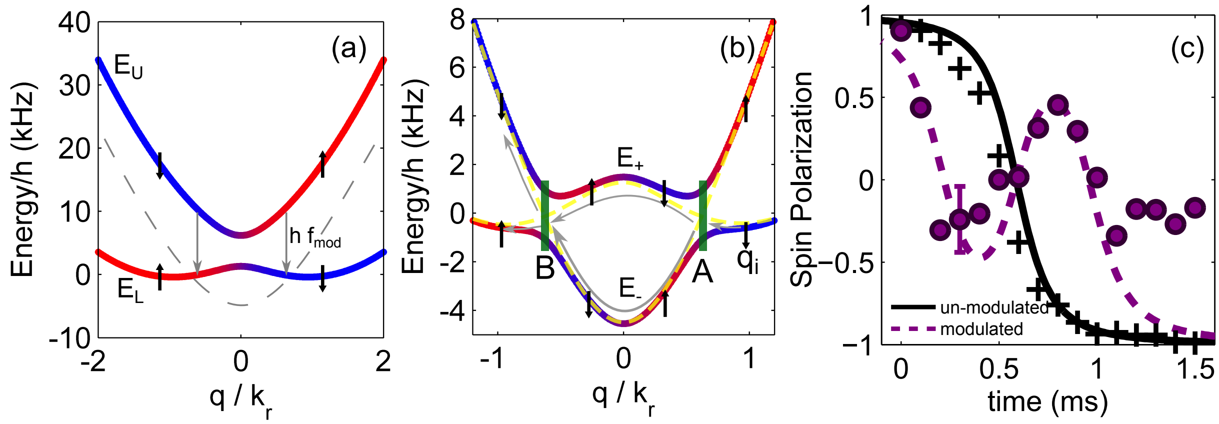

FIG. 1. (a) The standard Rashba-Dresselhaus SO eigenlevels created via Raman coupling of two atomic m_F ground states, here with ^{87}Rb and $\Omega_0 = 1.3E_r$, $\delta = 0$. The dashed line shows the upper eigenlevel E_U shifted down by the modulation frequency $f_{mod} = 10.56$ kHz. (b) Dressed-state diagram of the 3π SO-coupled eigenlevels created via modulation of the Raman coupling strength, which feature two avoided crossings (labeled A and B) here with the same parameters as (a) and $\Omega_C = 0.27E_r$. The modulation-dressed upper eigenlevel is denoted E_+, and the lower E_-, with Rashba-Dresselhaus eigenlevels shown by dashed yellow lines. The green bars indicated the avoided crossings used as beam splitters (in this case approximately 50/50) in the interference experiments. In (a) and (b) the blue and red colors represent spin down ($m_F = 0$) and up ($m_F = -1$) respectively. (c) Measurements of the spin-polarization of a BEC for transport in both the un-modulated system shown in (a) and the modulated system in (b). The BEC starts at q_i, labeled in (b), and falls under gravity ($\alpha_F = 1680\ k_r/s$) along the $-q$ direction and nearly adiabatically follows the lowest energy eigenlevel. With no modulation, the full π spin rotation is observed (black crosses). With modulation of $\Omega_M = 1.3E_r$, a nearly 3π rotation is observed (purple circles) as the quasimomentum, $\hbar q$, of the BEC goes from $+\hbar k_r$ to $-\hbar k_r$. Solid black (purple dashed) lines are the calculated spin polarization of E_L (E_-) given the same parameters as in experiment and $\Omega_C = 0.58E_r$ (calculated eigenlevel structure for this experiment is shown in Fig. 5). For this and following figures, a representative error bar indicates an average of 10% uncertainty in atom population in each spin due to technical noise.

Solving at each q, we obtain the new modulation-dressed eigenlevels, labeled $E_+(q)$ and $E_-(q)$, as eigenlevels of the Hamiltonian

$$\mathcal{H}_{3\pi} = \begin{pmatrix} E_{L,n=0}(q) & \Omega_C/2 \\ \Omega_C/2 & E_{U,n=-1}(q) \end{pmatrix}. \tag{2}$$

These eigenlevels feature a double avoided crossing with gap size Ω_C. Both $E_+(q)$ and $E_-(q)$ feature a 3π rotation of the atom-state spin polarization as the quasimomentum goes from $+\hbar q$ to $-\hbar q$, in contrast to the 1π rotation in the original E_U and E_L. Transport of a BEC in the new eigenlevels $E_+(q)$ and $E_-(q)$ are basis of our experimental investigations and are used to observe both 3π rotation of the spin and to engineer an atom interferometer.

For our experiments, we utilize transport methods developed in our earlier work [17] to study these modulation-dressed spin-orbit eigenlevels. Briefly, the initial state of the ^{87}Rb BEC is prepared by starting with a spin-pure BEC in the $m_F = 0$ state, and adiabatically turning on the Raman coupling to a fixed value Ω_0. This loads the atoms at the initial value $q_i \approx 1k_R$. The modulation of the Raman beams is then turned on, the optical trap holding the BEC is turned off, and then a force is applied in the $-q$ direction to accelerate the BEC at an average rate α_F through both the avoided crossings, labeled 'A' and 'B' in Fig 1. The probability of transition to the upper E_+ eigenlevel is determined by the Landau-Zener (LZ) probability $P_{LZ} = \exp\left[-2\pi(\Omega_C/2)^2/(\hbar\alpha\beta)\right]$ where $\alpha = |dq/dt|$ is the rate of acceleration at the avoided crossing and β is the difference of the slopes of the bare state energy levels [17]. The BEC is imaged after 15 ms time-of-flight expansion with a Stern-Gerlach field applied to measure both the spin and momentum of the BEC. The $|\downarrow\rangle$ ($|\uparrow\rangle$) state corresponds to ^{87}Rb hyperfine state $|F = 1, m_F = 0(-1)\rangle$. The recoil energy from the Raman lasers is $E_R = \hbar^2 k_r^2/2m = h \times 3.68$ kHz.

Fig. 1 (c) shows the experimentally measured spin polarization of the BEC accelerated by the gravitational force ($\alpha_F = 1680\ k_r/\mathrm{s}$) along the ground dressed eigenlevel. For $\Omega_M = 0$, the measured BEC spin polarization (black crosses) follows the calculated spin polarization of $E_L(q)$ (black line). For the case of strong modulation ($\Omega_M = 1.3E_r$) the BEC nearly adiabatically follows the calculated spin polarization of $E_-(q)$. The 1π spin rotation of E_L is evident in the data with no modulation, and

a 3π spin rotation is observed in the data with modulation. The measured spin polarization of the BEC doesn't perfectly match the calculated E_- eigenlevel spin polarization, as seen in the data after about 1 ms, due to imperfect loading into the dressed state and non-adiabatic inter-eigenlevel transitions [17]. Nonetheless, this experiment demonstrates the viability of modulated-Raman coupling as a means to create a new type of SO coupling with a 3π spin rotation, different from the previously studied, static Raman-induced SO with 1π spin rotation.

In addition to realizing a new type of 3π SO coupling, we use these modulated light induced synthetic gauge fields to engineer an atom-interferometer. Stueckelberg interference results when a quantum state is coherently split by LZ transition at an avoided-crossing, each split state travels along a different path, and then the states are recombined after another LZ transition at the same or a different avoided crossing [19, 20]. The final state depends on the energy difference of the two paths and the time it takes to traverse the path (together determining the total phase difference), and fringes in measurements of the final state will occur if the path or time is varied. The eigenlevels created using a traditional Raman coupling of internal atomic spin states, e.g. Fig. 1(a), do not give a usable pair of avoided crossings [21]. However, in the modulated system investigated here, such a pair is readily realized, and an example is indicated by vertical lines A and B in Fig. 1(b).

For these experiments, the energy difference is that between the upper and lower eigenlevels E_+ and E_-, and the BEC splits first at quasimomentum q_A and then is recombined at q_B with both processes involving Landua-Zener transitions ($P_{LZ}(\alpha) = \exp\left[-2\pi(\Omega_C/2)^2/(\hbar\alpha\beta)\right]$). The rate of acceleration, α, determines the time it takes to get from q_A to q_B, and the total phase difference acquired is calculated $\Phi(\alpha) = \int_{q_A}^{q_B}\left[E_+(q) - E_-(q)\right] dq/(\hbar\alpha)$. For the above and following equations, the dependence on Ω_0, Ω_C, δ, and f_{mod} has been suppressed for notational clarity. The output spin polarization, $\mathcal{S}$, for an atom moving through this interferometer with acceleration α is calculated as (see Appendix A):

$$\mathcal{S}(\alpha) = 4\left(P_{LZ}(\alpha) - P_{LZ}(\alpha)^2\right)\cos\left(\Phi(\alpha)\right) - (1-2P_{LZ}(\alpha))^2 \tag{3}$$

We measured Stueckelberg interference fringes using eigenlevel structures similar to that shown in Fig. 2(a). Here the energy difference of the two paths was controlled by f_{mod}, which in effect changes the length of the interferometer paths in q-space and thus the phase difference accumulated by the BEC components which travel along E_+ and E_-. Fig. 2 (b) shows the measured spin polarization of the BEC after it has passed both avoided crossings, and labeled are the calculated phase difference reaching 2π, 4π and 6π. Similar experiments were run for two values of Ω_0, and the diagram Fig. 2(a) shows that for smaller Ω_0, there is a greater energy separations of the two eigenlevels as observed in the Stueckelberg fringes dependence on f_{mod}.

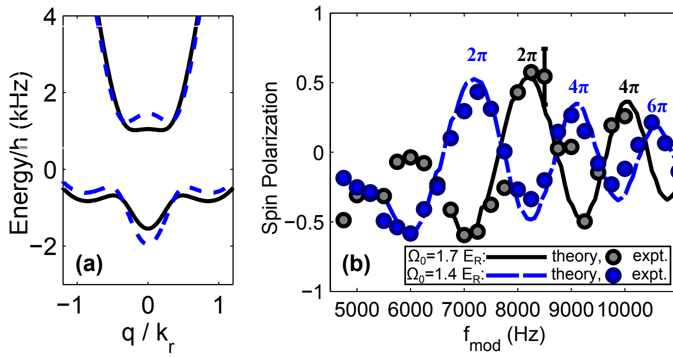

FIG. 2. (a) Two representative modulation induced 3π spin-orbit eigenlevels used to measure Stueckelberg interference with $\Omega_0 = 1.4E_r$ (blue dashed line) and $1.7E_r$ (black solid line), and $\delta = 0$. The eigenlevels are calculated using $\Omega_C = 0.3E_r$ and $f_{mod} = 8$ kHz for both. (b) Measured Stueckelberg interference fringes for $\Omega_0 = 1.4E_r$ (blue) and $1.7E_r$ (black), with $\Omega_M = 0.7E_r$, and $0.8E_r$ respectively and $\alpha_F = 1680k_r/s$ for both. For the calculated curves, $\Omega_C = 0.3E_r$, $\sigma_\alpha = 0.07\alpha_F$, $f_{np} = 0.4$, and other parameters match that of experiment.

In our experiments, a fraction of the ultracold atoms, denoted by f_{np}, do not participate in the Stueckelberg interference due to non-adiabatic initial state preparation. In addition, since the BEC is released from its confining potential at the start of transport it experiences a non-uniform acceleration due to atom-atom interactions about the central acceleration, α_F, created by the applied force. The non-uniform acceleration is modeled by assuming a Gaussian BEC acceleration distribution of $n(\alpha) = \frac{1}{\sqrt{2\pi}\sigma_\alpha}\exp\left[-(\alpha-\alpha_F)^2/(2\sigma_\alpha^2)\right]$. The values used in this paper for σ_α are consistent with numerically calculated solutions of the Gross-Pitaevski equation using a variational method with Gaussian ansatz and parameters similar to these experiments [22]. Accounting for $n(\alpha)$ and the non-participating fraction, the total spin polarization is calculated:

$$S_{tot} = (1 - f_{np})\int n(\alpha)\mathcal{S}(\alpha)d\alpha \tag{4}$$

Including both these effects, we obtain excellent agree-

ment with the experiment. This agreement is particularly notable given the period of the modulation is approximately only an order of magnitude shorter than the duration of the experiment, but verifies a time-averaged, dressed-state analysis for the eigenlevels can be used to model our experiments.

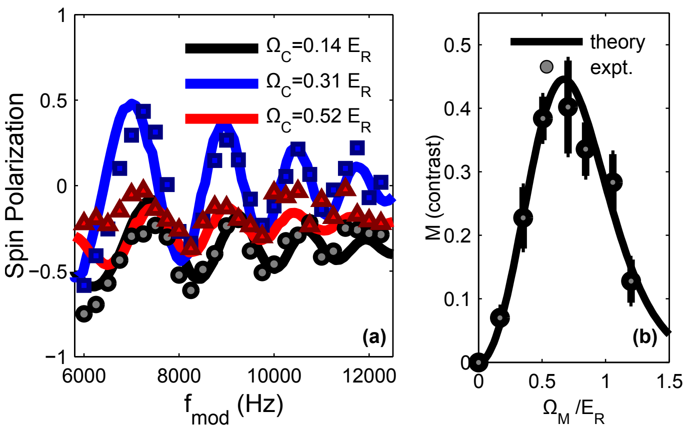

FIG. 3. (a) Observed Stueckelberg interference fringes for different Ω_M. Data (shown in the order of blue squares, red triangles and black circles) were taken with $\Omega_0 = 1.26, 1.40, 1.26E_r$, $\delta = 0E_r$, $\Omega_M = 0.34, 0.70, 1.2E_r$, and $\alpha_F = 1680k_r/s$. Theory fits are shown with $\sigma_\alpha = 0.07\alpha_F$, $\Omega_C = 0.14, 0.31, 0.52E_r$, $f_{np} = 0.4, 0.45, 0.5$. (b) Measured fringe contrast M versus modulation amplitude Ω_M. Changing Ω_M controls the probability to make a Landau-Zener transition between E_- and E_+, and thus fraction of the BEC that splits into each leg of the interferometer. Maximum contrast is expected when $P_{LZ} = 0.5$. The theory curve was generated assuming a linear relation of $\Omega_C = \Omega_M/2.25$, which was fit empirically from the data. For the theory, $\Omega_0 = 1.4E_r$, $f_{np} = 0.4$, $\sigma_a = 0.07\alpha_F$. Error bars indicate numerical fitting uncertainty of the fringes.

The gap size, Ω_C at the pair of avoided crossings affects the probability to travel along either eigenlevel, and thus controls the contrast of the Stueckelberg interference. Fig. 3 (a) shows Stueckelberg interference for a few representative values of Ω_M. Defining contrast as $M = (\mathcal{S}_{max} - \mathcal{S}_{min})/2$, Fig. 3 (b) shows the contrast observed for various Ω_M. The results show how Ω_M can be used to control the Landua-Zener transitions and thus the interference fringe amplitude. Given that the maximum contrast is obtained when the BEC is coherently split with $P_{LZ} = 0.5$, and assuming a linear relation between Ω_C and Ω_M, we find $\Omega_C = \Omega_M/2.25$ as the best fit for these parameters.

Fig. 4 shows Stueckelberg interference where the acceleration α_F of the BEC, rather than f_{mod}, is varied. A secondary optical dipole trapping force was used in combination with gravity to control the total acceleration [23] of the BEC and vary the time to transverse the two energy paths. Accelerations at which the calculated phase accumulations reach 4π and 6π are labeled. With the smaller acceleration, the effect of broadening in acceleration, σ_α, due to interactions reduces the observed fringe contrast near phase accumulation of 6π compared to 4π, but the fringes are still apparent and the model agrees well with the experimental results.

In summary, we have experimentally demonstrated using SO coupled BECs with modulated Raman coupling strength as a method to realize 3π SO coupling and engineer an atomic interferometer. Our theoretical analysis matches the results for measured Stueckelberg interference fringes, and confirms the treatment of the periodic modulation as "dressing" the SO coupled BEC states. Interestingly, since the SO coupling is itself the result of dressing the m_F free-particle dispersion with a Raman coupling, this can be considered as "dressing" dressed states. These initial experiments show the promise of 3π SO coupling, which could offer new opportunities for studying ground state and phase transitions in a 3-minima SO coupled BEC [2, 24]. These also demonstrate a different way from other recent studies involving periodic modulation of Raman coupling [25, 26] to engineer novel light-induced synthetic gauge fields.

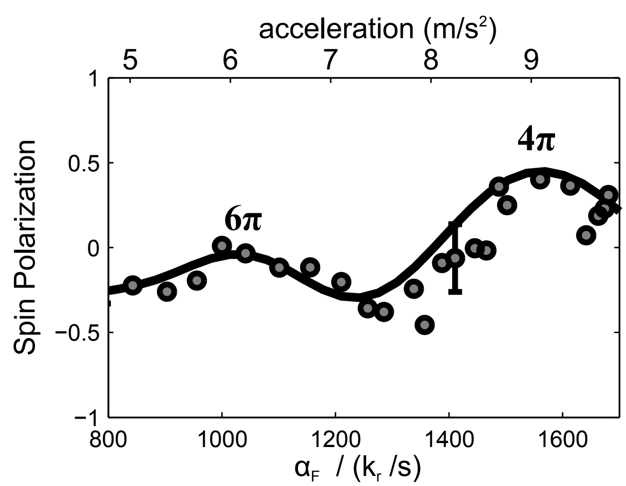

FIG. 4. Stueckelberg fringes in which the acceleration α_F is varied. The fringe contrast is strongly reduced at smaller α_F as the relative effect of σ_α gets larger with longer total time. $\Omega_0 = 1.40E_r$, $f_{mod} = 8.5$ kHz, $\Omega_M = 0.71E_r$, $\delta = 0$, and for theory plots $\Omega_C = 0.33E_r$, $\sigma_\alpha = 0.07 \times (1680\ k_r/s)$, and $f_{np} = 0.3$.

ACKNOWLEDGMENTS

The research was supported in part by DURIP-ARO Grant No. W911NF-08-1-0265, the Miller Family Endowment, and a Purdue University OVPR Research Incentive Grant. A.J.O. also acknowledges support of the U.S. National Science Foundation Graduate Research Fellowship Program. We thank Chris Greene, Li You, and Babak Seradjeh for helpful discussions.

Appendix A: Interferometry theory

We use the matrix method to solve for the BEC eigenlevel population resulting from the BEC splitting, phase accumulation, and recombination. $|\psi_\pm\rangle$ indicates the wavefunction in the $E_\pm$ eigenlevels respectively, so the state of the BEC is expressed $|\psi\rangle = c_+|\psi_+\rangle + c_-|\psi_-\rangle$. In operator notation, the state of the BEC is expressed

$$|\psi\rangle = \begin{pmatrix} c_+ \\ c_- \end{pmatrix}, \tag{5}$$

and the beam splitters take the form

$$\hat{B}_A = \begin{pmatrix} -\sqrt{1-P_{LZ}} & \sqrt{P_{LZ}} \\ \sqrt{P_{LZ}} & \sqrt{1-P_{LZ}} \end{pmatrix} \tag{6}$$

$$\hat{B}_B = \begin{pmatrix} \sqrt{1-P_{LZ}} & \sqrt{P_{LZ}} \\ \sqrt{P_{LZ}} & -\sqrt{1-P_{LZ}} \end{pmatrix} \tag{7}$$

in which P_{LZ} is the probability to make a diabatic transition in the modulation-dressed eigenlevels across the avoided crossing, and the negative signs on the diagonals account for phase shifts on the wavefunctions at each beam splitter [27]. The phase difference accumulated by the components of the BEC can be accounted for by a phase operator defined:

$$\hat{\Phi}(\phi) = \begin{pmatrix} e^{i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{pmatrix} \tag{8}$$

where ϕ is the phase difference accumulated. Readout of the final state composition is done by Stern-Gerlach separation of the bare-$|m_F\rangle$ states when the BEC has crossed both A and B at a point when the $E_\pm$ eigenlevels match the bare states to better than 97%, so that the spin polarization$= (N_{m_F=0} - N_{m_F=-1})/(N_{m_F=0} + N_{m_F=-1}) \approx (N_{|+\rangle} - N_{|-\rangle})/(N_{|+\rangle} + N_{|-\rangle})$. Thus, the readout of the spin polarization is given by

$$\hat{S} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{9}$$

The final state after the beam splitter A, phase operator, and beam splitter B is thus $|\psi_f\rangle = \hat{B}_B\hat{\Phi}\hat{B}_A|\psi_i\rangle$. The spin polarization is read $\langle\psi_f|\hat{S}|\psi_f\rangle$, and when solved with $|\psi_i\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ results in Eqn. 3.

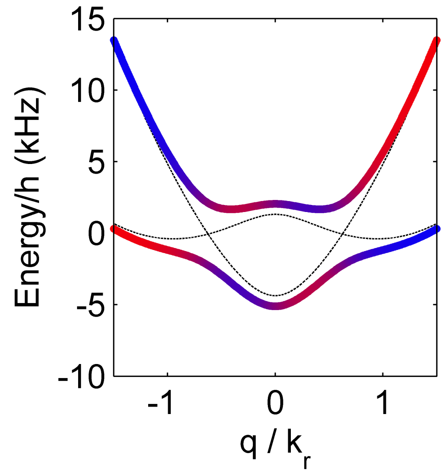

FIG. 5. Modulation dressed eigenlevel structure which possesses a 3π rotation of the spin polarization in the eigenlevels. Parameters are the same as used in the experiment shown in Fig.1 (c), with $\Omega_0 = 1.3E_r$, $\Omega_C = 0.58E_r$, $f_{mod} = 10.56$ kHz, and $\delta = 0$.

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